MONTHLY PROBLEMS IN MATHEMATICS

May 2024 Problems

1. Let S be the set of positive integers whose only prime factors are 2, 3, or 5. Evaluate

$$\sum_{x \in S} \frac{1}{x} = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10} + \frac{1}{12} + \frac{1}{15} + \ldots$$

2. Let B be a bounded closed convex symmetric (with respect to the origin) set in R^2 with boundary the curve Γ. Let B have the property that the ellipse of maximal area contained in B is the disc D of radius 1 centered at the origin with boundary the circle C. Prove that $A \cap \Gamma \neq \varnothing$ for any arc A of C of length $l(A) \geq \frac{\pi}{2}$.

3. Let S be the set of 3 by 3 symmetric matrices all of whose entries are 0 or 1. Find the number of matrices in S for which five entries are 1 and four are 0.

4. Prove that the arithmetic mean of all positive divisors of a positive integer n lies in theinterval $\left[\sqrt{n}, \frac{n+1}{2}\right]$.

(*Problems 2, 3, 4 are proposed by Henry Ricardo*)

5. (a) For a real number a, and integer $n > 1$, if possible, find functions f and g, each having at least n nonconstant derivatives, such that a is a zero of f, g and all their non constant derivatives.
(b) Do the same if f and g are required to be polynomials of degree $n+1, n \geq 1$.
(*Proposed by Mahmoud Sayrafiezadeh*)

MEC Monthly Problems in Mathematics
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Deadline for submitting solutions is September 30
Please send solutions to Mah_Sayr@icloud.com
Please type solutions in Word
with equations in Mathtype

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