MONTHLY PROBLEMS IN MATHEMATICS

April 2024 Problems

1. We throw a die (which selects one of the numbers 1, 2, …, 6 with equal probability) n times. What is the probability that the sum of the values is divisible by 5?

2. Let all roots of an nth degree polynomial $P(z)$ with complex coefficients lie on the unit circle in the complex plane. Prove that all roots of the polynomial $2zP'(z) - nP(z)$ lie on the same circle.

3. Let $\left\{ \varepsilon_n \right\}_{n=1}^{\infty}$ be a sequence of positive real numbers such that $\lim_{n \to \infty} \varepsilon_n = 0$. Find

$$\lim_{n \to \infty} \frac{1}{n} \sum_{k=1}^{n} \ln\left(\frac{k}{n} + \varepsilon_n \right).$$

4. Let V be a real vector space, and let $f, f_1, f_2, \ldots, f_k$ be linear maps from V to $\mathbb{R}$. Suppose that $f(x) = 0$ whenever $f_1(x) = f_2(x) = \cdots = f_k(x) = 0$. Prove that f is a linear combination of $f_1, f_2, \ldots, f_k$.

(*Above problems are proposed by Henry Ricardo*)

5. An experiment consists of throwing n fair coins and recording the product of the number of heads by the number of tails that come up.
(a) What is the probability that the expected value of the outcome will be a whole number?
(b) For what values of n does the expected value of the outcome coincide with one of the actual outcomes?
 (*Proposed by Mahmoud Sayrafiezadeh*)