



MONTHLY PROBLEMS IN MATHEMATICS

April 2024 Problems

1. We throw a die (which selects one of the numbers 1, 2, ..., 6 with equal probability) n times. What is the probability that the sum of the values is divisible by 5?
2. Let all roots of an n th degree polynomial $P(z)$ with complex coefficients lie on the unit circle in the complex plane. Prove that all roots of the polynomial $2zP'(z) - nP(z)$ lie on the same circle.
3. Let $\{\varepsilon_n\}_{n=1}^{\infty}$ be a sequence of positive real numbers such that $\lim_{n \rightarrow \infty} \varepsilon_n = 0$. Find

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \ln \left(\frac{k}{n} + \varepsilon_n \right).$$

4. Let V be a real vector space, and let f, f_1, f_2, \dots, f_k be linear maps from V to \mathbb{R} . Suppose that $f(x) = 0$ whenever $f_1(x) = f_2(x) = \dots = f_k(x) = 0$. Prove that f is a linear combination of f_1, f_2, \dots, f_k .

(Above problems are proposed by Henry Ricardo)

5. An experiment consists of throwing n fair coins and recording the product of the number of heads by the number of tails that come up.
 - (a) What is the probability that the expected value of the outcome will be a whole number?
 - (b) For what values of n does the expected value of the outcome coincide with one of the actual outcomes?

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MEC Monthly Problems in Mathematics
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Deadline for submitting solutions is April 30
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Please type solutions in Word
with equations in Mathtype

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